Tracking the Moon's Altitude with a Quadrant and Triquetrum

Zorina Ahmad Syed

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Abstract

Centuries before the telescope, astronomers fixed the positions of celestial objects with nothing more than a weighted string, a sighted ruler, and a graduated scale. This study revisits two such instruments, the quadrant and the triquetrum, to ask a simple question: how well can naked-eye, pre-telescopic technology track a moving target like the Moon? Over thirteen consecutive nights (18–30 June 2026), the Moon's altitude was measured once nightly with both instruments and cross-checked against the modern planetarium software Stellarium. After propagating Type B measurement uncertainties through each instrument's geometry, the quadrant tracked the Moon to within $-0.25° \pm 0.15°$ of the software reference and the triquetrum to within $-0.08° \pm 0.06°$. Both comfortably resolved the Moon's day-on-day rise toward, and just past, its peak altitude on 27 June. The results show that simple analog instruments, even when built from acrylic and aluminium rather than brass, can reproduce a modern ephemeris to a few tenths of a degree. They also show that the triquetrum's cosine-rule readout is the more precise of the two designs.

1 Introduction

Long before satellites and planetarium software could tell us exactly where to look, the night sky was read with the naked eye and a handful of mechanical instruments. The altitude of a celestial object, its angular height above the horizon, was one of the most fundamental quantities an observer could measure, and an entire family of instruments was built around measuring it: the astrolabe, the armillary sphere, the quadrant, and the triquetrum (also called the parallactic ruler) [3]. Of these, the quadrant and the triquetrum are some of the oldest and conceptually simplest. They trace back to the 2nd-century astronomer Claudius Ptolemy and were refined over the following millennium by astronomers across the Islamic world and Renaissance Europe [4].

This period of refinement was not incidental. Muslim scholars between the 8th and 15th centuries had a direct, daily motivation for precise altitude measurement: determining the times of the five daily prayers and the direction of the Qibla. This need drove sustained innovation in instrumentation at major observatories such as Maragha in present-day Iran [5] and, later, the short-lived but ambitious Istanbul Observatory of Taqi al-Din [6]. These efforts produced theoretical and practical advances in instrument design, in spherical trigonometry, and in observational technique. These advances would later inform European astronomers, including Copernicus, who is known to have drawn on the parallactic-ruler tradition described in Book IV of *De revolutionibus* [8, 7].

The triquetrum used in the present study is itself a direct descendant of this tradition: its triangular geometry follows the same isosceles-arm design as a 1948 reconstruction held at the Nicolaus Copernicus Museum in Frombork, Poland [10]. The quadrant, in turn, follows the layout of an early-19th-century mariner's quadrant held at the National Maritime Museum, Greenwich [9].

Most demonstrations of these instruments stop at observing the Sun. This study instead applies them to the Moon, a target whose altitude changes measurably from one night to the next as it moves through its ∼27.3-day sidereal orbit. This makes it a more demanding test of an instrument's day-to-day repeatability than a single afternoon of solar tracking. The aim was twofold: first, to determine whether the quadrant and triquetrum could resolve the Moon's nightly altitude trend at all; and second, to quantify exactly how precise each instrument is, using a full Type B uncertainty propagation, against the reference values produced by Stellarium.

This choice of target carries a further significance beyond instrument testing. The Islamic (Hijri) calendar is a purely lunar calendar: each of its twelve months begins not on a fixed date but with the sighting of the young lunar crescent, or *hilal*, low above the western horizon shortly after sunset [11]. Determining whether the crescent is visible on a given evening is itself an altitude problem: the Moon must clear a minimum altitude above the horizon, against a still-bright twilight sky, before it can be sighted by eye. The low-altitude readings at the start of this data set (Table 1) are therefore a direct analogue of the observation historically used to open a new Hijri month. By coincidence, the present thirteen-night window falls almost exactly across such a transition: 1 Muharram 1448 AH, the Islamic New Year, corresponds to 16 June 2026 on the Umm al-Qura calendar [11], one day before this study's observations begin. The tenth of Muharram, the Day of Āshūrā, falls on 25 June 2026, squarely inside the observation period. Table 1 therefore doubles as a small illustration of the same crescent-visibility geometry that has governed the start of the Islamic calendar for over fourteen centuries.

2 Equipment

The quadrant and triquetrum used in this study (Fig. 1) were designed, built, and provided by Mirza Sarim Ahmed Beg, Khadim Mehmood, and Dr. Muhammad Sabieh Anwar of PhysLab, Lahore University of Management Sciences (LUMS), as part of the Lahore Science Foundry's Phenomenon Lab outreach series [1, 2]. The quadrant is a white acrylic sector with a 0°–90° scale (least count 0.1°) and a plumb line hanging from its vertex. The triquetrum is a metal-and-aluminium isosceles frame with two fixed arms of length $L = 93\,\mathrm{cm}$, a sliding sight on the ruler arm, and a tripod stand. Both instruments were used exactly as supplied, without modification.

3 Operating Procedure

Both instruments were set up side by side and read once nightly at 22:52 local time, following the procedure described in the accompanying activity manual [1].

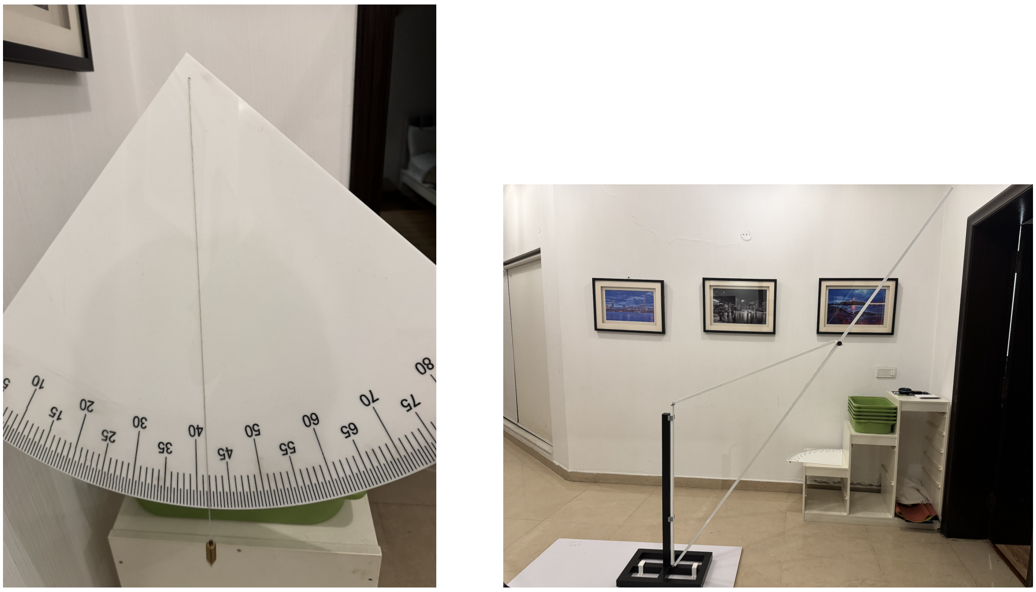
Quadrant

1. **Aim:** sight along the quadrant's edge, where the 0° mark begins, directly at the Moon, keeping the instrument steady.

2. **Record:** let the plumb line hang freely and read the value where it crosses the 0°–90° scale. This is the zenith distance Z.

3. **Calculate:** subtract the reading from 90° to obtain the altitude,

$$\theta_Q = 90^\circ - Z. \tag{1}$$



(a) Quadrant (b) Triquetrum

Figure 1: The instruments used in this study, photographed by the author. Both were designed and built by Mirza Sarim Ahmed Beg and colleagues at PhysLab, LUMS [1, 2].

Triquetrum

1. **Sight:** look from the bottom hinge along the ruler arm toward the Moon.
2. **Align:** slide the moving arm's slider until it lines up exactly with the target, then lock it in place.
3. **Measure:** record the value R (cm) where the slider rests on the ruler arm.
4. **Solve:** the triquetrum forms an isosceles triangle, so the cosine rule for the zenith distance simplifies to

$$Z = \arccos\left(\frac{R}{2L}\right), \qquad L = 93\,\text{cm}, \tag{2}$$

5. **Final altitude:** subtract Z from 90°, giving $\theta_T = 90^\circ - Z$.

On 18 and 19 June, cloud cover obscured the Moon entirely and no reading could be taken with either instrument (recorded as N/A in Table 1).

4 Uncertainty Analysis

Both instruments are analog, so every reading carries a Type B uncertainty, following the same convention used in the reference PhysLab study [2]:

$$\sigma = \frac{\Delta}{\sqrt{6}}, \qquad \Delta = \tfrac{1}{2}(\text{least count}), \tag{3}$$

the standard uncertainty for a reading assumed uniformly distributed within one least count.

Quadrant. The scale has a least count of 0.1°, so $\Delta = 0.05^\circ$ and, from Eq. (3),

$$\sigma_{\theta_Q} = \frac{0.05^\circ}{\sqrt{6}} \approx 0.02^\circ. \tag{4}$$

Triquetrum. The ruler-arm scale has a least count of 0.1 cm, giving $\sigma_R \approx 0.02$ cm; the same precision is assumed for the fixed arm length, $\sigma_L \approx 0.02$ cm. Because θ_T depends on R and L through Eq. (2), these two uncertainties must be propagated through the zenith-distance formula via standard error propagation:

$$\sigma_Z = \sqrt{\left(\frac{\partial Z}{\partial R}\sigma_R\right)^2 + \left(\frac{\partial Z}{\partial L}\sigma_L\right)^2}. \tag{5}$$

Since $Z = \arccos(u)$ with $u = R/2L$, the partial derivatives must be found using the chain rule together with the standard derivative $\frac{d}{du}\arccos(u) = -\frac{1}{\sqrt{1-u^2}}$, rather than treated as an ordinary polynomial or trigonometric derivative:

$$\frac{\partial Z}{\partial R} = \frac{d}{du}\Big[\arccos(u)\Big] \cdot \frac{\partial u}{\partial R} = -\frac{1}{\sqrt{1-u^2}} \cdot \frac{1}{2L}, \tag{6}$$

$$\frac{\partial Z}{\partial L} = \frac{d}{du}\Big[\arccos(u)\Big] \cdot \frac{\partial u}{\partial L} = -\frac{1}{\sqrt{1-u^2}} \cdot \left(-\frac{R}{2L^2}\right) = \frac{R}{2L^2\sqrt{1-u^2}}. \tag{7}$$

Substituting $u = R/2L$ gives $\sqrt{1-u^2} = \sqrt{1-(R/2L)^2} = \dfrac{\sqrt{4L^2-R^2}}{2L}$, which simplifies the two derivatives to

$$\frac{\partial Z}{\partial R} = -\frac{1}{\sqrt{4L^2-R^2}}, \qquad \frac{\partial Z}{\partial L} = \frac{R}{L\sqrt{4L^2-R^2}}. \tag{8}$$

Since $\theta_T = 90^\circ - Z$ is simply the complement of Z, its uncertainty is unchanged: $\sigma_{\theta_T} = \sigma_Z$.

Evaluating Eqs. (5) and (8) across the data set (Table 1) gives σ_{θ_T} between 0.006° and 0.010°, roughly an order of magnitude smaller than the quadrant's uncertainty. This is what sets the number of significant figures quoted in Table 1: θ_Q is reported to one decimal place (matching its 0.1° least count) and θ_T to two decimal places.

To express a 95% confidence interval on the graph in Fig. 2, both uncertainties were doubled (expanded uncertainty), following the same convention as [2].

5 Results

Across the eleven usable nights, the quadrant deviated from Stellarium by $-0.25^\circ \pm 0.15^\circ$ (mean ± standard deviation) and the triquetrum by $-0.08^\circ \pm 0.06^\circ$. There was no systematic zero offset of the kind reported for earlier wood-stand triquetrum builds [2].

6 Discussion

Trend. The Moon's altitude (Fig. 2) rises steadily from 5.6° on 20 June to a peak of about 30° on 27 June, then falls to 26.0° by 30 June. This is the lunar analogue of the Sun's seasonal altitude cycle, compressed into the Moon's ∼27.3-day sidereal orbit. As the Moon moves along its orbit (inclined ∼5° to the ecliptic and further modulated by its own changing declination), its altitude at a fixed clock time and location rises and falls over the course of roughly two weeks. It reaches a maximum near the point where the Moon crosses closest to the observer's meridian for that part of the cycle. Because only thirteen consecutive nights were sampled, only the rising half and the very start of the falling half of this longer cycle are visible here. A full two-week-plus dataset would be expected to show the altitude continue falling before turning upward again as the cycle repeats. It is worth noting that the lowest readings in this data set (20 June, 5.6°) were taken only four nights after the new moon that opened the current Hijri month (1 Muharram 1448 AH, 16 June 2026). The crescent-sighting observation that historically marks the start of an Islamic month is made at a comparably low altitude, just above the horizon in evening twilight, a few nights before the Moon reaches even this height.

Table 1: Lunar altitude tracking log, 18–30 June 2026, readings taken nightly at 22:52. Hijri dates follow the Umm al-Qura calendar [11]; local moon-sighting authorities may observe a date one day later.

Date	Hijri date	θ_Q (°)	R (cm)	θ_T (°)	Stellarium (°)
18/6	3 Muharram 1448	N/A	N/A	N/A	N/A
19/6	4 Muharram 1448	N/A	N/A	N/A	N/A
20/6	5 Muharram 1448	5.2 ± 0.02	18.1 ± 0.02	5.61 ± 0.01	5.57
21/6	6 Muharram 1448	11.0 ± 0.02	35.5 ± 0.02	11.07 ± 0.01	11.24
22/6	7 Muharram 1448	16.2 ± 0.02	52.1 ± 0.02	16.36 ± 0.01	16.48
23/6	8 Muharram 1448	20.4 ± 0.02	65.8 ± 0.02	20.72 ± 0.01	20.83
24/6	9 Muharram 1448	24.3 ± 0.02	77.4 ± 0.02	24.56 ± 0.01	24.65
25/6	10 Muharram 1448 (Āshūrā)	27.1 ± 0.02	85.3 ± 0.02	27.30 ± 0.01	27.34
26/6	11 Muharram 1448	28.9 ± 0.02	90.3 ± 0.02	29.08 ± 0.01	29.19
27/6	12 Muharram 1448	29.7 ± 0.02	92.9 ± 0.02	29.91 ± 0.01	30.01
28/6	13 Muharram 1448	29.4 ± 0.02	92.1 ± 0.02	29.58 ± 0.01	29.71
29/6	14 Muharram 1448	28.4 ± 0.02	89.2 ± 0.02	28.34 ± 0.01	28.35
30/6	15 Muharram 1448	26.1 ± 0.02	81.8 ± 0.02	26.03 ± 0.01	26.04

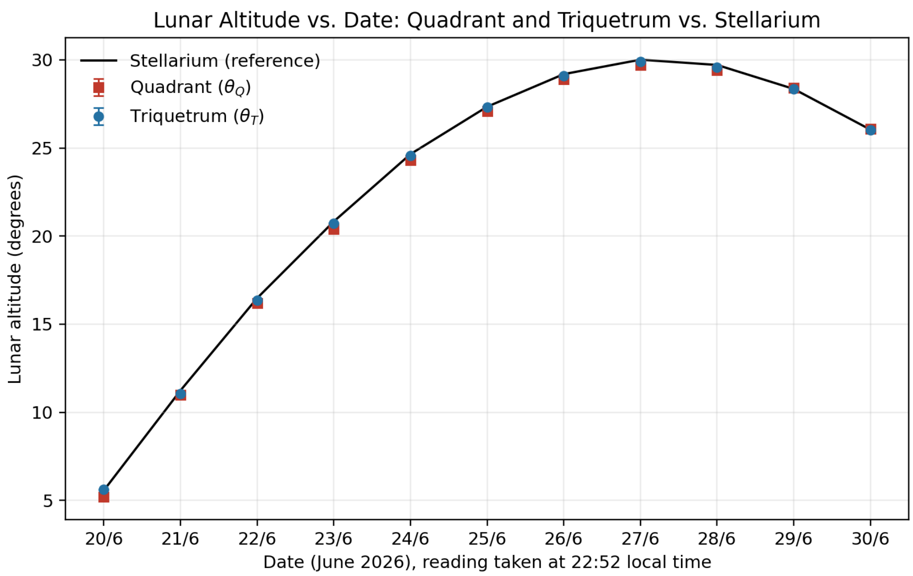


Figure 2: Lunar altitude vs. date for the quadrant, triquetrum, and Stellarium reference. Error bars show expanded (∼95% confidence) uncertainties; for the triquetrum these are smaller than the marker size.

Instrument performance. Both instruments track the reference curve closely, but the triquetrum is consistently closer to Stellarium and has a propagated uncertainty roughly 2–3 times smaller than the quadrant's reading uncertainty. This matches expectation: the triquetrum's cosine-rule readout amplifies a small linear-scale reading error far less than the quadrant's direct angular scale does at these altitudes. In addition, the rigid metal construction used here avoided the wooden-stand flexing (and resulting $\sim 6^\circ$ zero offset) reported in an earlier solar trial with a wood-framed triquetrum [2].

7 Conclusion

Over an eleven-night observation window, both the quadrant and the triquetrum successfully reproduced the Moon's rise-then-fall altitude trend, with deviations from Stellarium of 0.25° and 0.08° respectively, well within the precision expected of naked-eye medieval instruments. The results confirm that these instruments remain effective, low-cost tools for introducing observational astronomy and for demonstrating, very concretely, how pre-telescopic astronomers tracked the changing positions of celestial objects.

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