How does a bottle empty?

بوتل کیسے خالی ہوتی ہے؟

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Have you ever observed water pour from a hole in a bottle—smooth, steady, and shimmering in the light, yet curiously thinner than the opening it flows through? Around 1643, Italian physicist Evangelista Torricelli beautifully established the relationship between efflux speed and water height. Yet his model, elegant in its simplicity, described an ideal fluid—one without loss, without contraction, without the quiet friction of reality.

In this experiment, you will not only demonstrate Torricelli's insight, but also extend it—universalizing the law so it holds true for any container, large or small, and any fluid, water or otherwise. So let's get started.

کیا آپ نے کبھی کسی بوتل کے سوراخ سے پانی کو بہتے ہوئے دیکھا ہے—ہموار، مسلسل اور روشنی میں چمکتا ہوا—مگر حیرت انگیز طور پر اس سوراخ سے بھی باریک جس سے وہ نکل رہا ہوتا ہے؟ تقریباً 1643 میں اطالوی طبیعیات دان ایوانجلیستا ٹوریچیلی نے پانی کے اخراج کی رفتار اور اس کی سطحی بلندی کے درمیان تعلق کو نہایت خوبصورتی سے واضح کیا۔ تاہم، ان کا یہ ماڈل اپنی سادگی میں جتنا دلکش تھا، اتنا ہی مثالی بھی تھا۔ یہ ایک ایسے سیال کی وضاحت کرتا تھا جس میں نہ کوئی توانائی کا نقصان ہو، نہ بہاؤ کا سکڑاؤ، اور نہ ہی حقیقت کی وہ خاموش رگڑ جو عملی دنیا میں موجود ہوتی ہے۔

اس تجربے میں آپ نہ صرف ٹوریچیلی کی بصیرت کو عملی طور پر دکھائیں گے بلکہ اسے مزید وسعت بھی دیں گے—قانون کو اس حد تک عالمگیر بنا دیں گے کہ وہ ہر قسم کے برتن، بڑے ہوں یا چھوٹے، اور ہر سیال، چاہے پانی ہو یا کوئی اور مادہ، پر درست ثابت ہو۔ تو آئیے، آغاز کرتے ہیں۔

Let's make sure we have everything we need to make our setup which is shown in Fig. 1.

1. Graduated cylinder with orifice, along with a water guide
2. PhysLoad (to measure mass)
3. PhysLogger (to bring data into the computer)
4. Water container
5. Vernier caliper

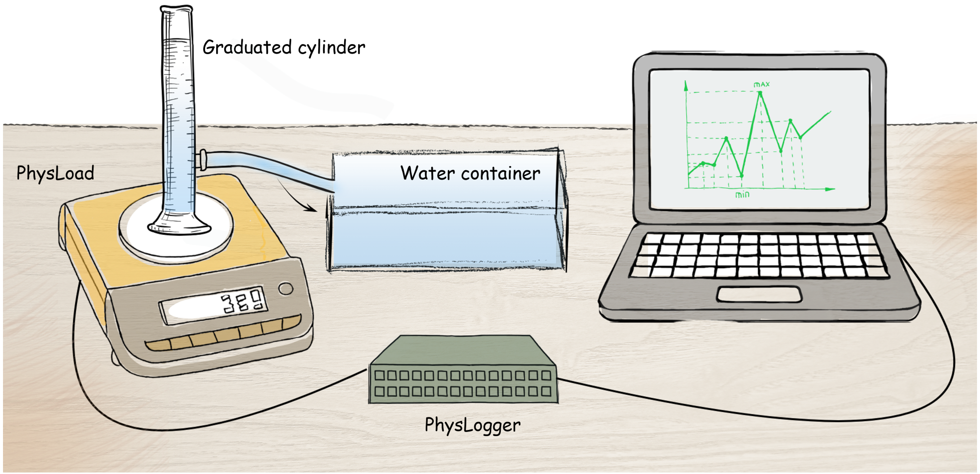
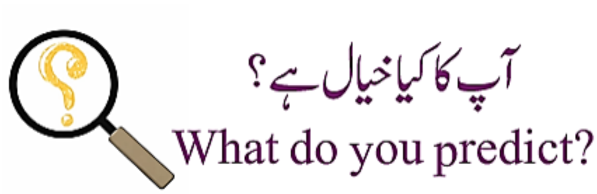
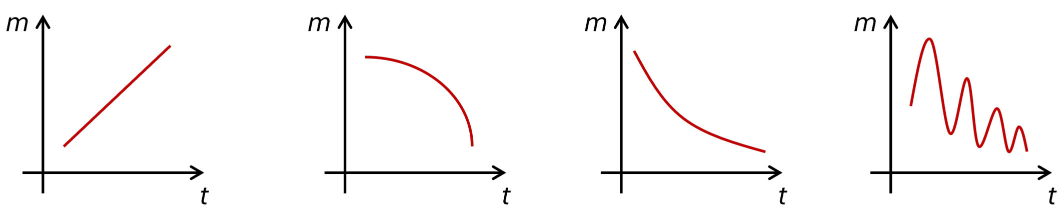


Figure 1: A graduated cylinder filled with water is placed on a PhysLoad, an a computer-interfaced mass balance. Water ejects out from a orifice, and follows a path through a water guide into a collection container. The PhysLogger records the decrease in water mass in the cylinder over time.

As the cylinder initially filled with water empties, because there is an orifice on its side, the mass of the water decrease with time. This is what gets into the computer.



[Q 1]. Make a sketch of how you predict the mass (m) will decrease with time (t). Is your answer one of the following? Or any thing else.



Set up the experiment by placing the empty graduated cylinder onto the PhysLoad. Connect it to the PhysLogger and use the PhysLogger desktop software to tare it (see Appendix). Next, fill the cylinder with water up to your chosen starting height. Position the collection container so that the water stream flowing from the pinhole lands cleanly inside it. Finally, begin data collection with PhysLogger, open the pinhole, and allow the water to drain out.

Measure the diameter of water cylinder and orifice using a vernier caliper to obtain their cross-sectional area. Write it down in your notebook.

ڈیٹا کیا کہانی سناتا ہے؟

What does the data say?

To find how the cylinder empties, we need to know the speed at which it discharges from the orifice. Consider Fig. 2. Water flows out from a narrow circular nozzle at a fixed height h_2 from the base. The nozzle has a small area A_{nozzle} compared to the cross-sectional area $A_{cylinder}$ of the cylinder. As time progresses, the level of the water $h_1(t)$ in the cylinder descends and water issues out with a speed $v_2(t)$.

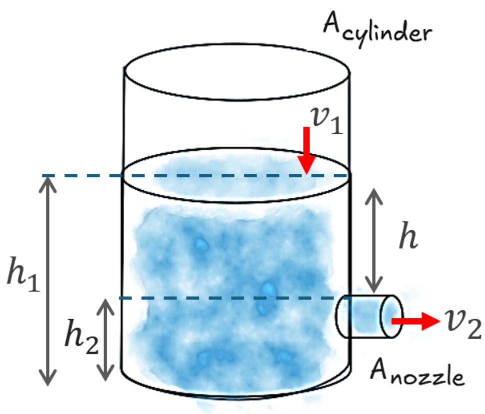
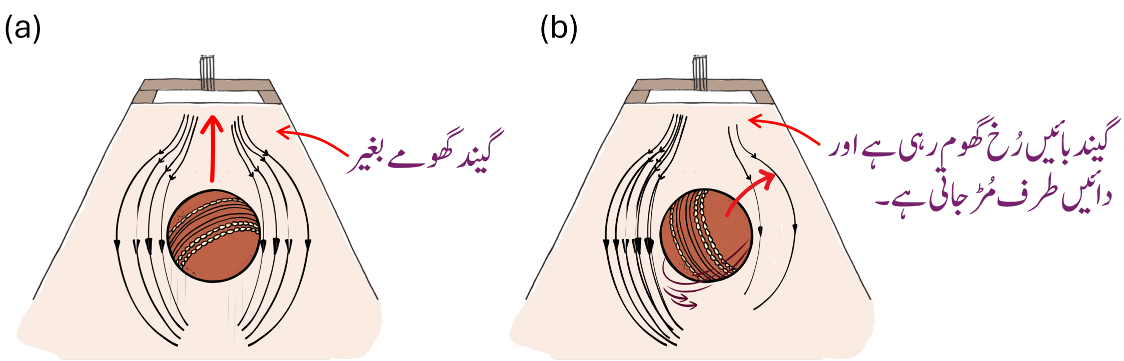


Figure 2: A cylinder with water flowing out from a narrow orifice at a fixed height h_2 from the base.

In order to find $v_2(t)$, we use a principle proposed by the famous Bernoulli, which in short is that, higher the speed, lower the pressure.

وسیم اکرم سوئنگ کا بادشاہ



تصویر (b) میں جب گیند بائیں جانب گھومتی ہے تو بائیں سمت مساوی دباؤ کے خط گنجان ہو جاتے ہیں، یعنی ایک دوسرے کے بالکل قریب آجاتے ہیں، جس کے نتیجے میں بائیں طرف دباؤ میں اضافہ اور دائیں طرف دباؤ میں کمی واقع ہوتی ہے۔ اس کے نتیجے میں گیند دائیں طرف مڑ جاتی ہے۔ اسے "اِن سوئنگ" کہتے ہیں۔

Mathematically, the relationship between pressure and speed at some height can be expressed as:

$$\begin{aligned} &\text{Pressure at height } 1 + \left[\frac{1}{2} \times \rho \times (\text{speed at height } 1)^2\right] + \left[\rho \times g \times (\text{height } 1)\right] \\ &= \text{Pressure at height } 2 + \left[\frac{1}{2} \times \rho \times (\text{speed at height } 2)^2\right] + \left[\rho \times g \times (\text{height } 2)\right] \end{aligned} \tag{1}$$

where ρ is the density of the medium and g is the acceleration due to gravity.

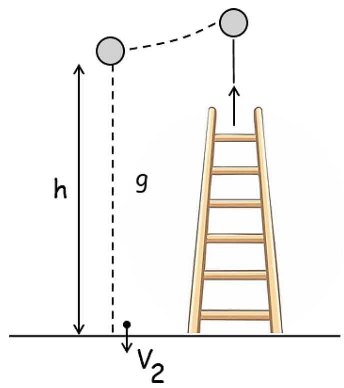
Now coming back to Fig. 2, P_o is the pressure at height 1 (h_1) which is equal to atmospheric pressure identical to the pressure at height 2 (h_2) since the orifice opens into the atmosphere. So let's write the Bernoulli principle given above as,

$$P_0 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_0 + \frac{1}{2}\rho v_2^2 + \rho g h_2, \tag{2}$$

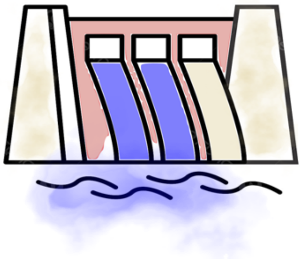
and since $A_{cylinder} \gg A_{nozzle}$, $v_1 \approx 0$, leading to

$$v_2^2 = 2gh. \tag{3}$$

Suppose an object is dropped from a height h. Its speed v_2 as it hits the ground will be $v_2^2 = 2gh$, which is exactly the same formula for the speed of ejecting liquid when the depth of water above it is h. See how different natural phenomena have marked similarities.



مزے کا سوال



ڈیم کے اسپل وے سے پانی تیزی سے باہر بہہ رہا ہے۔ اسپل وے کے دہانے سے نکلتے وقت پانی کی رفتار کیا ہو گی؟

Now let's turn to your data.

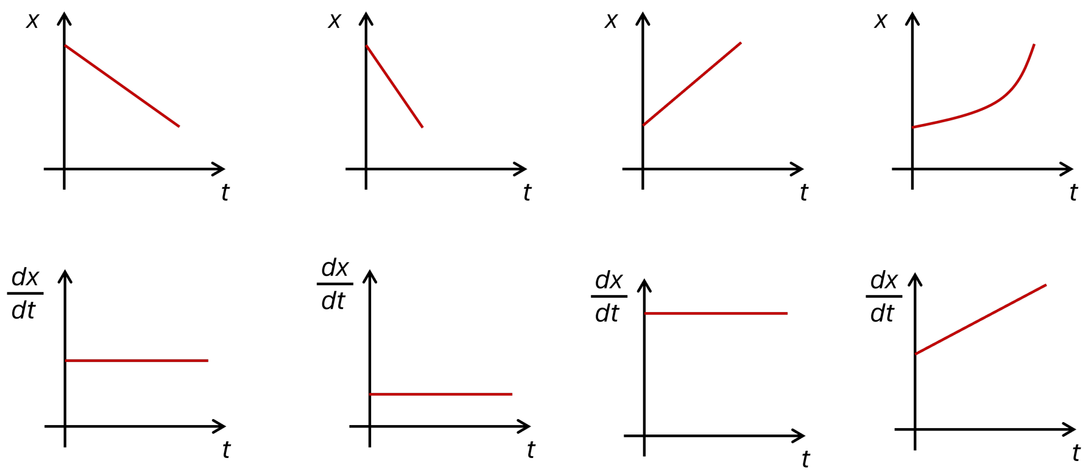
[Q 2]. Make a plot of $m_1(t)$ versus t. Here m_1 is the mass of the liquid in container.

[Q 3]. Make a plot of $m(t)$ versus t. Here m is the mass of the liquid in the container above the orifice.

[Q 4]. What is the relationship between $m(t)$ and $h(t)$? Plot $h(t)$ versus t.

We now turn to the idea of a 'derivative'. The derivative of a quantity tells us how much change in a certain quantity happens in a really small interval of time. So it is a measure of how fast or slow a quantity changes. Given below are some quantities and their derivatives, shown by the symbol

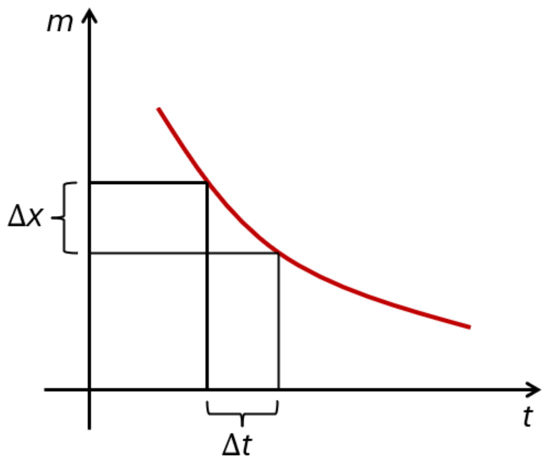
$$\frac{d(x)}{dt} = \text{derivative of } (x) \text{ with respect to time.}$$



From data, the process of finding the derivative is simple. You measure the change in one quantity over a change in time:

$$\frac{d(x)}{dt} = \frac{\Delta x}{\Delta t}.$$

So one way of finding the derivative is to take two successive points of the data set and divide by the time interval between those two successive points. Let's put this recipe in terms of symbols:



$$\frac{d(x_i)}{dt} = \frac{x_{i+1} - x_i}{t_{i+1} - t_i},$$

where x_{i+1} is the value of x at the time t_{i+1} and x_i at the time t_i. Another way to calculate the derivative which is more realistic is to use a 'three-point' derivative given by:

$$\frac{d(x_i)}{dt} = \frac{3x_{i+2} - 4x_{i+1} + x_i}{t_{i+2} - t_i}.$$

[Q 5]. Plot $\frac{dm(t)}{dt}$ versus t using:

(a) the single-point derivative,
(b) the three-point derivative. Compare (a) and (b), and comment which approximation is more appropriate.

[Q 6]. Plot $\frac{dh(t)}{dt}$ versus t using:

(a) the single-point derivative,
(b) the three-point derivative. Compare (a) and (b), and discuss which approximation is more appropriate.

[Q 7]. What is the relationship between v_2 and $\frac{dm(t)}{dt}$?

From Q4, your answer should be:

$$m = Ah\rho, \tag{4}$$

whereas from Q7, your answer should be:

$$v_2 = -\frac{1}{\rho A_{orifice}}\left(\frac{dm}{dt}\right). \tag{5}$$

Verify your answers and discuss with your instructor.

[Q 8]. In Q6, we plotted $h(t)$ versus t. We would like to see if this phenomenal observation can be explained through a model that describes the flow of a liquid. From Eqs. (1) and (2), and Toricelli's equation show that,

$$\frac{dh}{dt} = -2g\Big(\frac{A_{orifice}}{A_{cylinder}}\Big)\sqrt{h(t)}. \tag{6}$$

For simplification, let's write $\kappa = A_{orifice}/A_{cylinder}$, Eq. (3) become,

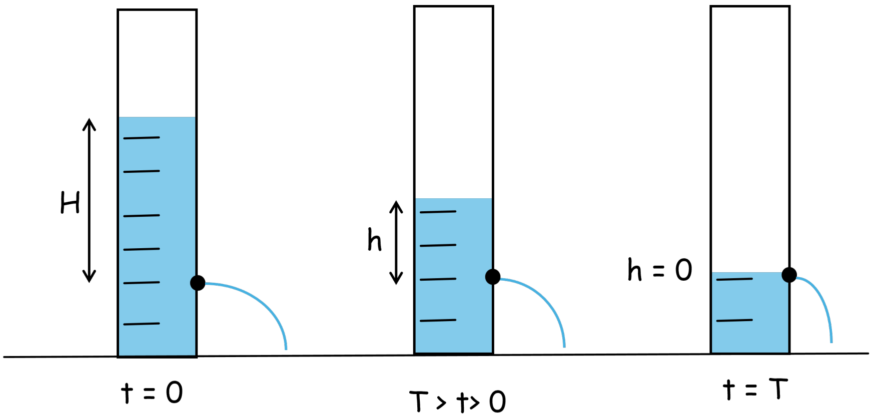
$$\frac{dh(t)}{dt} = -2g\kappa\sqrt{h(t)}. \tag{7}$$

This equation is an example of a differential equation. Solving it means that we can predict $h(t)$ versus t. This is our mathematical model and our intention is to check whether our data—coming out from the phenomenon under investigation—matches the model.

Talented students can show that when Eq. (7) is solved, the outcome is,

$$h(t) = \Big(\sqrt{H} - \kappa\sqrt{\frac{g}{2}}t\Big)^2, \tag{8}$$

where $H=$ height above the orifice at time $t = 0$.



In order to see if your data from Q3 fits Eq. (8), we need to know $\kappa = A_{orifice}/A_{cylinder}$. Unfortunately, κ is not known. So it appears to be really hard to validate if the model works. However, one last trick can be pulled out of our sleeves. From above Figure, we see that when $t = T$, the bottle empties, i.e. $h = 0$ which means that (From Eq. (8)),

$$\sqrt{H} = \kappa\sqrt{\frac{g}{2}}T. \tag{9}$$

Let's divide both sides of Eq. (8) by H:

$$\begin{aligned}\frac{h(t)}{H} &= \left(\frac{\sqrt{H} - \kappa\sqrt{\frac{g}{2}}\,t}{\sqrt{H}}\right)^2 \\ &= \left(1 - \frac{\cancel{\kappa\sqrt{\frac{g}{2}}}\,t}{\cancel{\kappa\sqrt{\frac{g}{2}}}\,T}\right)^2 \\ &= \left(1 - \frac{t}{T}\right)^2 \end{aligned} \tag{10}$$

where in the second last step, we have used Eq. (9). Putting $h(t)/H = h'$ and $t/T = \tau$, Eq. (10) then becomes,

$$h'(t) = (1 - \tau)^2. \tag{11}$$

This simple equation tells the full story of how the tank drains with time. Last, you can also derive efflux velocity for the present model by taking first derivative of Eq. (11).

$$v'(t) = \frac{dh'(t)}{dt} = 2(1 - \tau). \tag{12}$$

Essentially Eq . (11) and Eq. (12) are universal. The relationship does not change when data is collected with a wide or narrow graduated cylinder, a different size of orifice, or even a different liquid.

Table 1 and Table 2 highlights how the variables $h'(\tau)$ and $v'(\tau)$ plays out at specific moments in time.

Table 1: Normalized water height as a function of normalized time.

τ	$h'(\tau)$	Value	Explanation
0	$(1-0)^2$	1	Full tank
0.5	$(1-0.5)^2$	0.25	25% water remains
1	$(1-1)^2$	0	Empty tank

Table 2: Normalized efflux speed as a function of normalized time.

τ	$v'(\tau)$	Value	Explanation
0	$(1-0)$	1	Maximum speed
0.5	$(1-0.5)$	0.5	Speed reduced to 50%
1	$(1-1)$	0	No efflux

[Q 9]. Plot h' and v' as a function of τ. Does normalized experimental data follow normalized theoretical model provided in Eq. (11) and Eq. (12)? How do you account for any difference?

[Q 10]. Finally, is Eq. (3) satisfied? Can it be used to predict g? If not, why not?

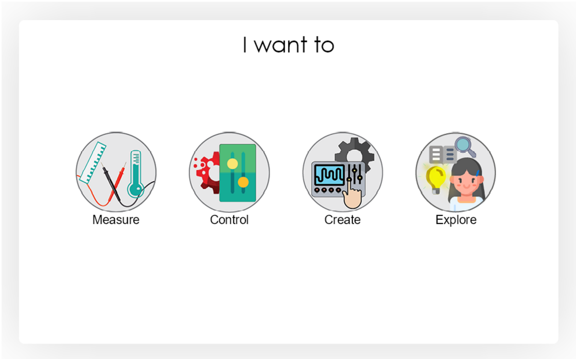
References

[1] P.M.C. de Oliveira, A. Delfino, E.V. Costa, and C.A.F. Leite, "Pin-hole water flow from cylindrical bottles," Phys. Educ. **35(2)**, (2000).

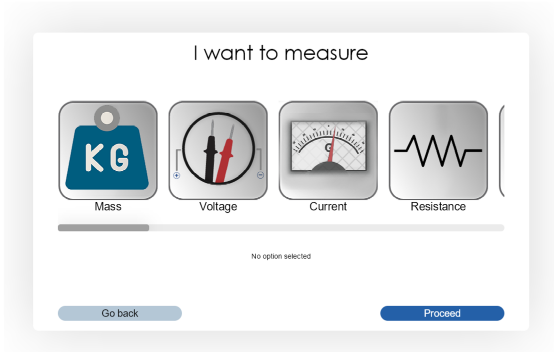
Appendix

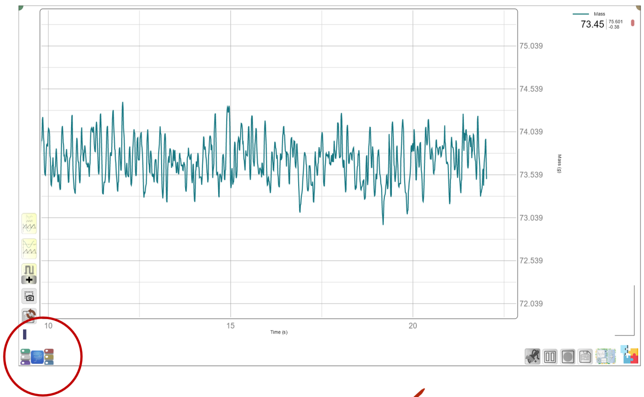
PhysLoad setting, and how to tare it?

In the PhysLogger desktop application , select **PhysLogger** and then click **Continue**. Go to **Measure**.

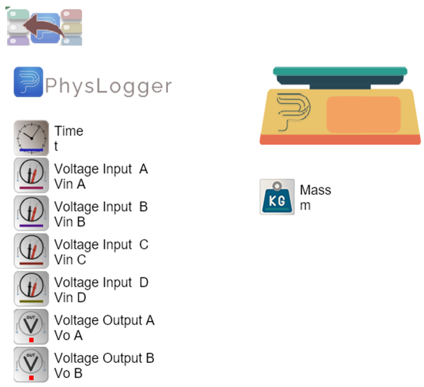


Select **Mass**, then **Proceed**, and Click on **Make a live plot now**.





Here you all set for data visualization using **PhysLoad**. Next pop-up **Quantities** menu at bottom left of your screen to tare the PhysLoad.



Select **Mass** to tare. Additionally, you can also change the sampling frequency by clicking on Time if required.